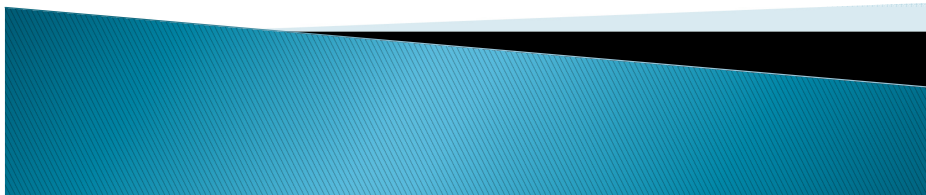


Dr. Öncü HAZIR

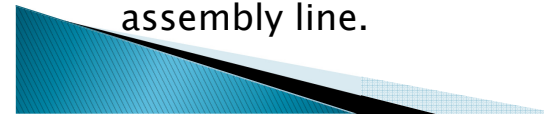
Project Management

Project Scheduling & Resource Planning



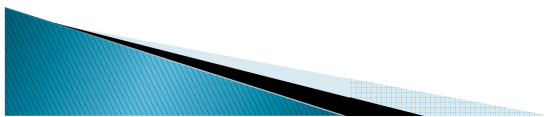
Reminding Question

- ▶ Identify the example that would not usually be considered a **project**.
 - a) Building a house
 - b) Developing a computer software application program
 - c) Hosting a wedding reception
 - d) Manufacturing automobiles on the assembly line.



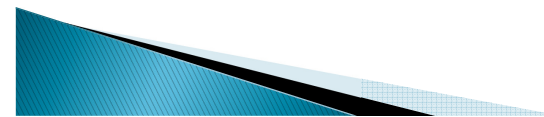
Reminding Question

- ▶ Prime **objectives** of **project management** are
 - a) Performance, cost, time
 - b) Quality, scope, schedule
 - c) Customer satisfaction, budget, schedule
 - d) Cost, quality, customer satisfaction



Reminding Question

- ▶ PERT stands for
 - a. Program Evaluation and Retrieval Technique.
 - b. Program Evaluation and Review Technology.
 - c. Program Evaluation and Review Technique.
 - d. Planning Evaluation and Review Technique.
 - e. Planning Evaluation and Retrieval Technique.



Steps of Project Planning

1. Develop statement of work
2. Define work breakdown structure
3. List resources needed
4. Estimate time required

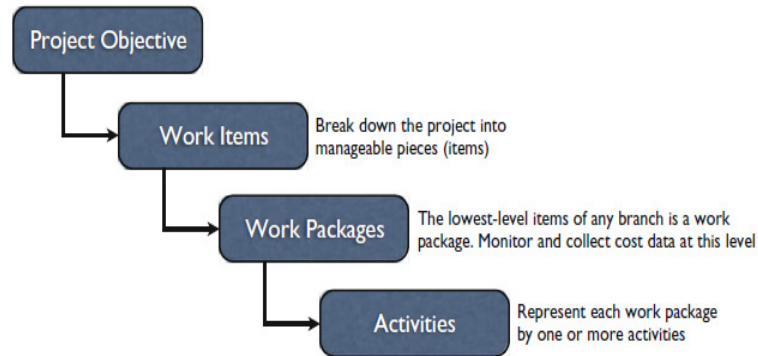
Vonderembse, Mark A., and Gregory P. White. *Core concepts of operations management*. John Wiley & Sons Inc, 2003.

Hierarchical Planning System

- All activities required to complete a project must be precisely delineated, and **coordinated**
- Some activities must be done **sequentially**, and some **simultaneously**
- Using a hierarchical planning system will allow these activities to be identified and sorted appropriately
- Also know as the “even planning process”

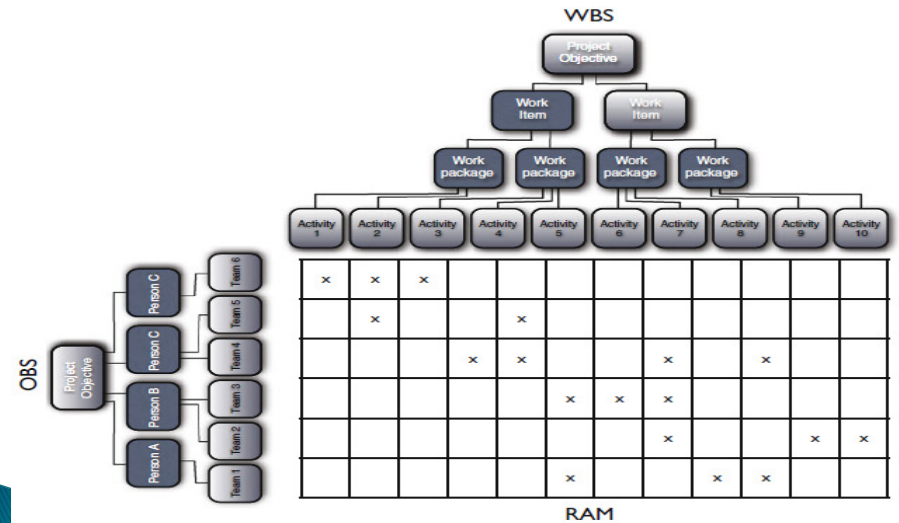
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Work Breakdown Structure (WBS)



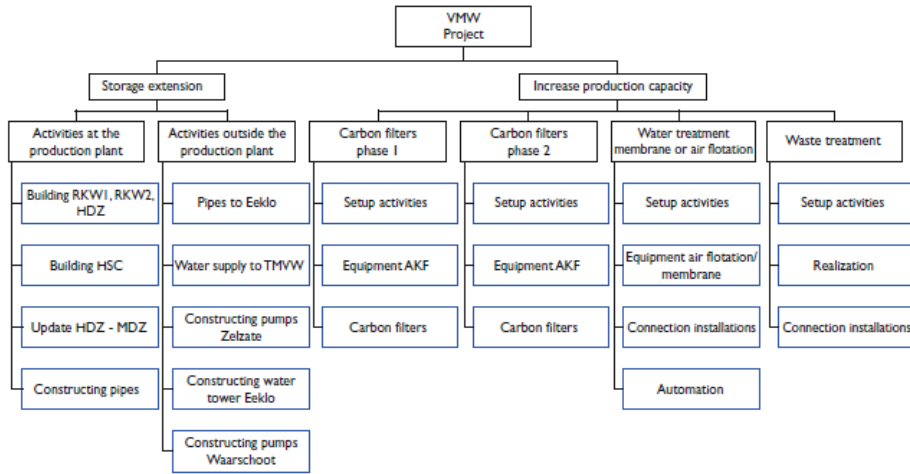
Vanhoucke, Mario. *Project management with dynamic scheduling*. Springer, Berlin, 2012.

Work Breakdown Structure (WBS)



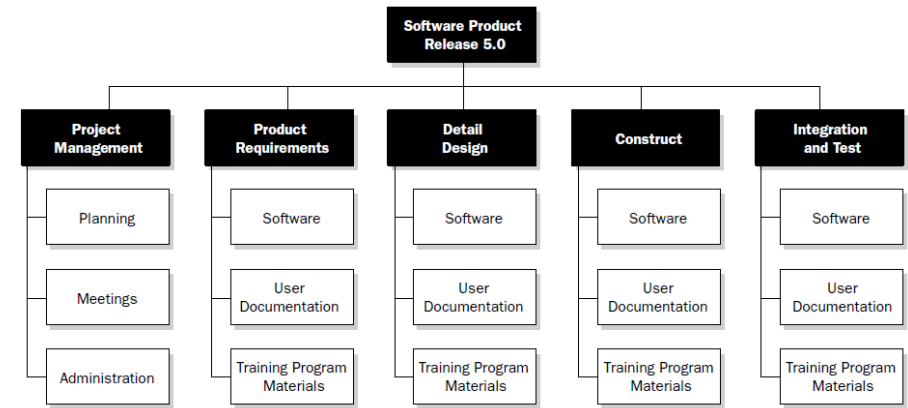
Vanhoucke, Mario. *Project management with dynamic scheduling*. Springer, Berlin, 2012.

WBS Example: Capacity expansion project at a water production center



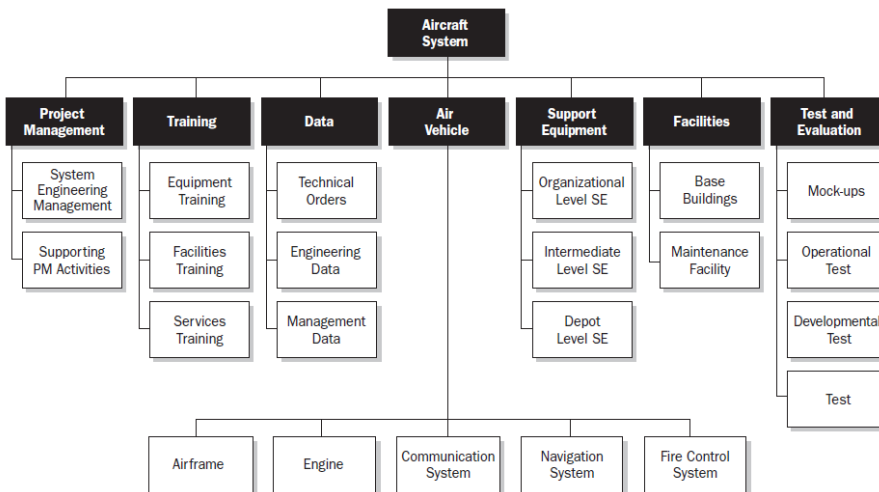
Vanhoucke, Mario. *Project management with dynamic scheduling*. Springer, Berlin, 2012.

WBS with Phases & Deliverables



PMBOK 5th Edition Project Management Institute.

WBS with Major Deliverables



PMBOK 5th Edition Project Management Institute.

Work Breakdown Structure (WBS)

- **Project objective:** The project objective consists of a short description of the scope of the project. A careful **scope definition** is of crucial importance in project management.
- **Work item:** Project is broken down into manageable pieces (items) to be able to cope with the **project complexity**.
- **Work package:** The monitoring and collection of cost data often occurs at this level.
- **Activity:** The **lowest level** of the **WBS**, where the accuracy of cost, duration and resource estimates can be improved, and where the precedence relations can be incorporated.

Vanhoucke, Mario. *Project management with dynamic scheduling*. Springer, Berlin, 2012.

Work Breakdown Structure (WBS)

In planning a project, the project manager must structure the **work into small elements** that are:

- ▶ **Manageable**, in that specific authority and responsibility can be assigned,
- ▶ Independent, or with **minimum interfacing** with and dependence on other ongoing elements,
- ▶ **Integratable** so that total package can be seen,
- ▶ **Measurable** in terms of progress,

Kerzner, Harold R. *Project management: a systems approach to planning, scheduling, and controlling*. John Wiley & Sons, 2013.

Work Breakdown Structure (WBS)

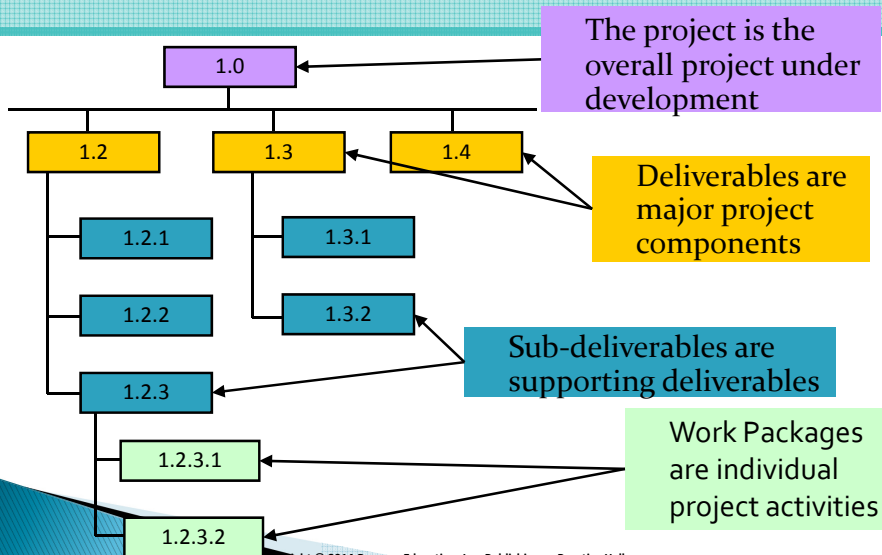
A process that sets a project's scope by **breaking down** its overall **mission into** a cohesive set of synchronous, increasingly **specific tasks**.

What does WBS accomplish?

- ❖ Echoes project objectives
- ❖ Offers a logical structure
- ❖ Establishes a method of control
- ❖ Communicates project status
- ❖ Improves communication
- ❖ Demonstrates control structure

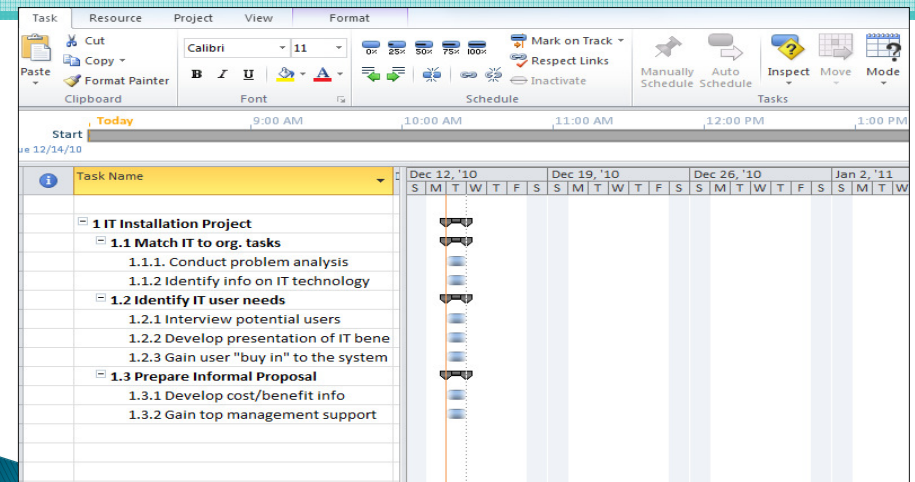
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Work Breakdown Structure and Codes



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Sample WBS in MS Project 2010



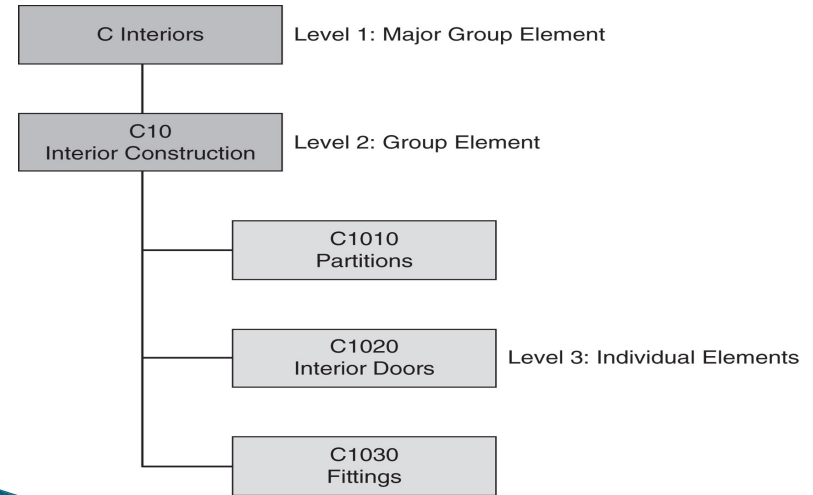
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WBS: Examples

| | |
|--|----------|
| Program: New Plant Construction and Start-up | 01-00-00 |
| Project 1: Analytical Study | 01-01-00 |
| Task 1: Marketing/Production Study | 01-01-01 |
| Task 2: Cost Effectiveness Analysis | 01-01-02 |
| Project 2: Design and Layout | 01-02-00 |
| Task 1: Product Processing Sketches | 01-02-01 |
| Task 2: Product Processing Blueprints | 01-02-02 |
| Project 3: Installation | 01-03-00 |
| Task 1: Fabrication | 01-03-01 |
| Task 2: Setup | 01-03-02 |
| Task 3: Testing and Run | 01-03-03 |
| Project 4: Program Support | 01-04-00 |
| Task 1: Management | 01-04-01 |
| Task 2: Purchasing Raw Materials | 01-04-02 |

Kerzner, Harold R. *Project management: a systems approach to planning, scheduling, and controlling*. John Wiley & Sons, 2013.

WBS in graphic form



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Defining a Project Work Package

1. Work package forms **lowest level** in WBS.
2. Work package has a **deliverable result**.
3. Work package may be considered by its owner as a project in itself.
4. A work package may include several **milestones**.
5. A work package should fit organizational **culture**.
6. The optimal size of a work package may be expressed in terms on **labor hours**, calendar time, cost, reporting period, and risks.

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Quick Question

- ▶ The desired outcomes or results of a project are called _____.
 - a) Subgroups
 - b) Work packages
 - c) Subprojects
 - d) Deliverables

Activities sequence

EXTERIOR FINISH ACTIVITIES SEQUENCE

1. Housewrap
2. Roofing
3. Masonry
4. Siding
5. Concrete Prep
6. Soffitt and fascia
7. Pour driveway and sidewalk
8. Exterior painting
9. Landscaping

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Project Scheduling Techniques

- ▶ Gantt chart
- ▶ Critical Path Method (CPM)
- ▶ Program Evaluation & Review Technique (PERT)
- ▶ Critical Chain Method
- ▶ Resource Constrained Project Scheduling
- ▶ Multi-Mode Project Scheduling

Gantt chart schedule for a house with a basement

| Month | January | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|-----------------------|---------|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|--|--|--|--|--|--|--|--|--|
| Calendar Date | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | | | | | | | | | |
| Construction Day | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | | | | | | | | | |
| Crew Prep | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Kickoff meeting | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Site Work | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Footings/Fdn. | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| WP Fdn. | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Sub Slab | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Bsmt. Slab | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Back Fill/Rough Grade | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Windows | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Exterior Concrete | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Roofing | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Masonry | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Rough Plumbing | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Rough HVAC | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Rough Electrical | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Wall Insulation | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Exterior Trim | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Drywall | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Exterior Paint | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

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Sample Gantt Chart

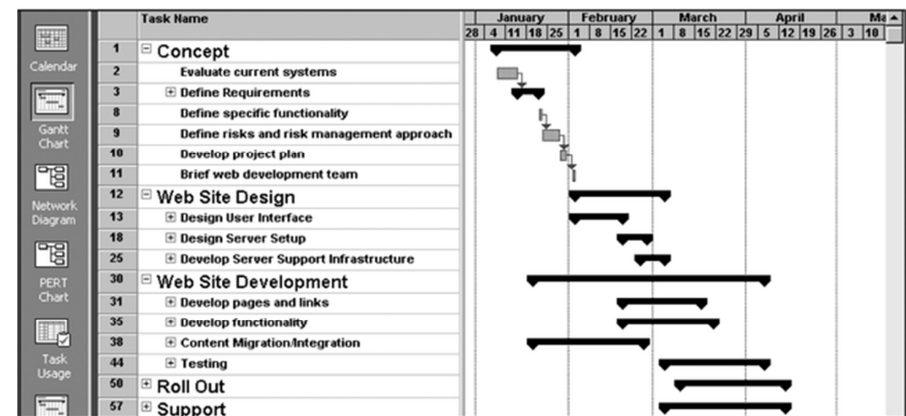


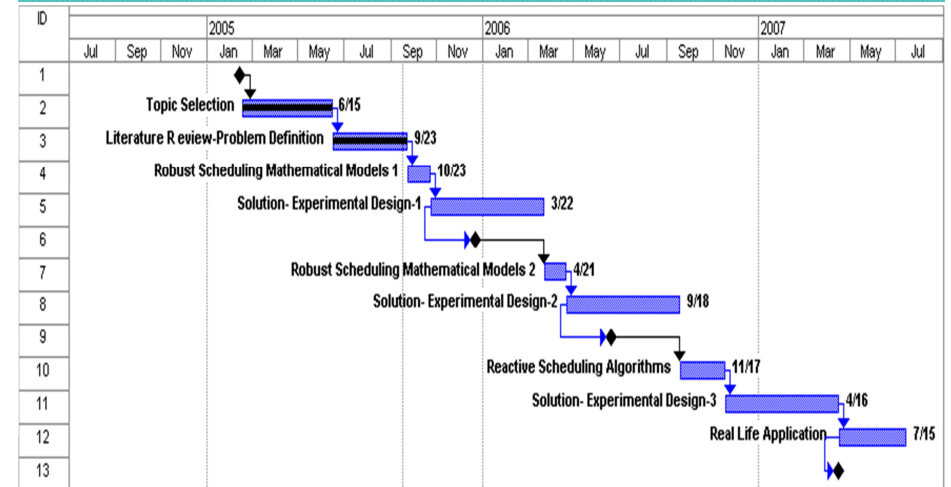
Figure 1-3. Sample Gantt Chart in Microsoft Project 2002

Mantel, Meredith, Shafer, and Sutton, Project Management in Practice, John Wiley & Sons,

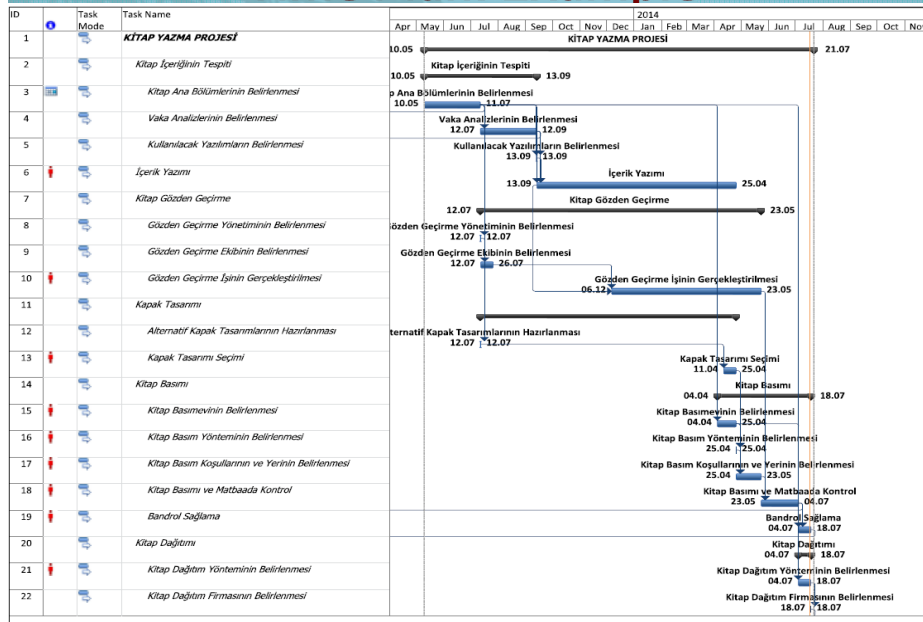
Time Plan

| Task Name | Duration (days) | Starting Time | Finish Time |
|--|-----------------|---------------|-------------|
| Qualifier Exam | 2 | Feb. 2005 | Feb. 2005 |
| Topic Selection | 120 | Feb. 2005 | Jun. 2005 |
| Literature Review & Problem Definition | 100 | Jun. 2005 | Sept. 2005 |
| Mathematical Models Robust Scheduling 1 | 30 | Sep. 2005 | Oct. 2005 |
| Solution- Experimental Design 1 | 150 | Oct. 2005 | Mar. 2006 |
| Working Paper 1 | 90 | Dec. 2005 | Mar. 2006 |
| Mathematical Models Robust Scheduling 2 | 30 | Mar. 2006 | Apr. 2006 |
| Solution- Experimental Design 2 | 150 | Apr. 2006 | Sept. 2006 |
| Working Paper 2 | 90 | Jun. 2006 | Sept. 2006 |
| Reactive Scheduling Algorithms | 60 | Sep. 2006 | Nov. 2006 |
| Experimental Design (Reactive Scheduling) | 150 | Nov. 2006 | Apr. 2007 |
| Real Life Application | 90 | Apr. 2007 | Jul. 2007 |
| Working Paper 3 | 90 | Apr. 2007 | Jul. 2007 |

Time Plan: Example



Time Plan: Example

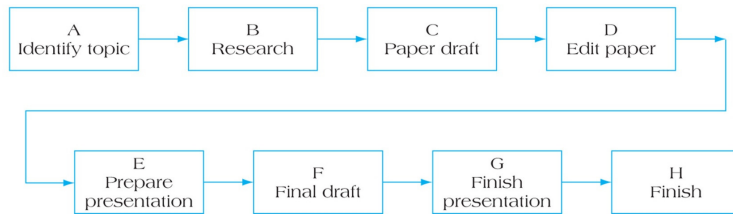


Project Scheduling

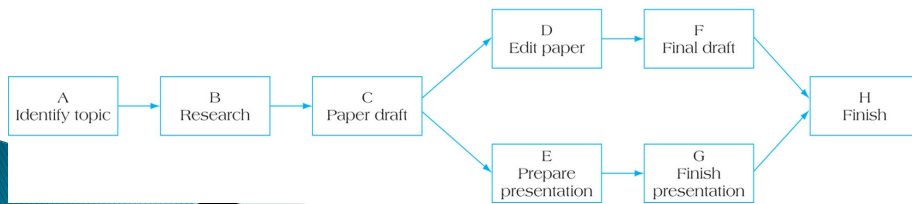
- Previously, projects had been managed using Gantt charts.
- The method developed by the U.S Navy was called the **Program Evaluation and Review Technique (PERT)**.
- Du Pont and Remington-Rand developed a technique that does not use probabilities and called it the **critical path method (CPM)**.

Network Diagrams

Option A: Serial Sequential Logic

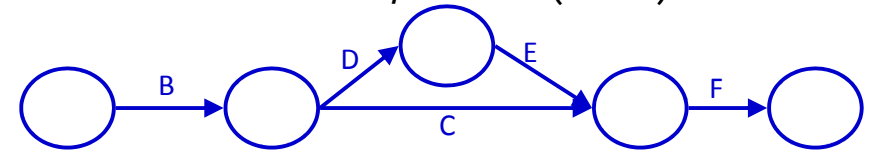


Option B: Nonserial Sequential Logic

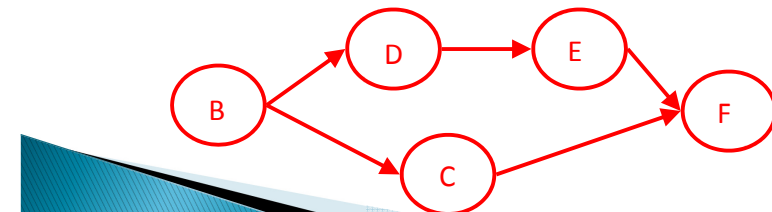


Network Representation of a Project

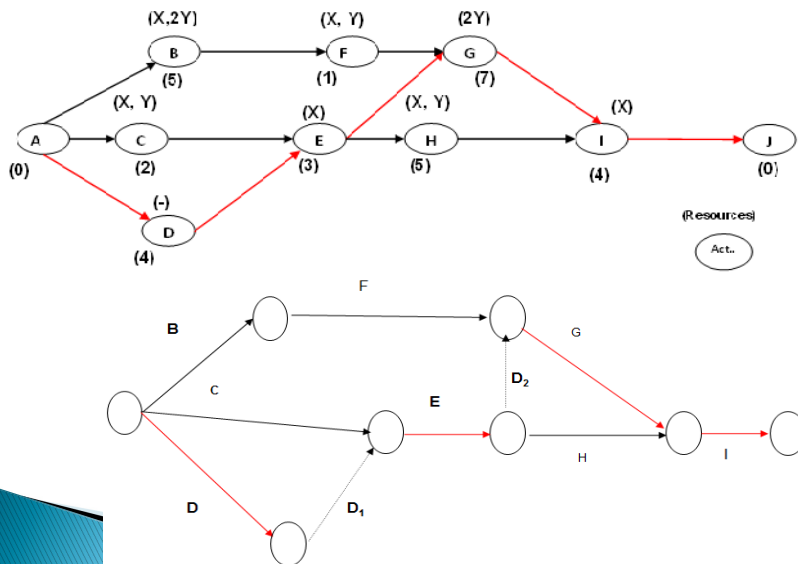
□ Activities on Arcs/Arrows (AOA)



□ Activities on Nodes (AON)



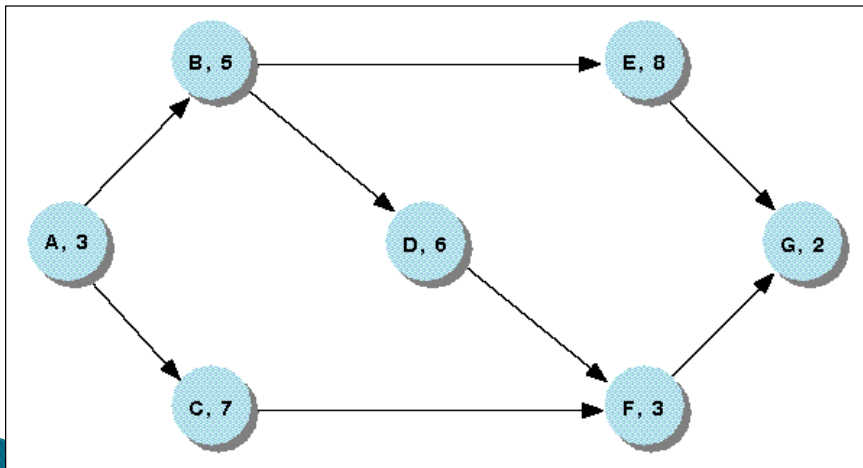
Network Diagrams (AON, AOA)



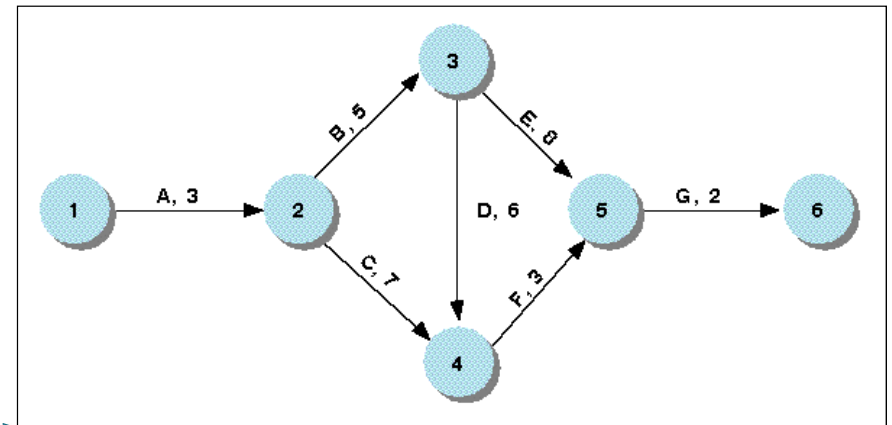
Precedence Relationships for Information System Project

| Activity | Most Likely Duration (Weeks) | Immediate Predecessor(s) |
|-----------------------------|------------------------------|--------------------------|
| A. Select computer software | 3 | — |
| B. Install software | 5 | A |
| C. Install office network | 7 | A |
| D. Test software | 6 | B |
| E. Develop database | 8 | B |
| F. Train employees | 3 | C, D |
| G. Implement system | 2 | E, F |

Activity-on-Node Network Representation of Information System Project

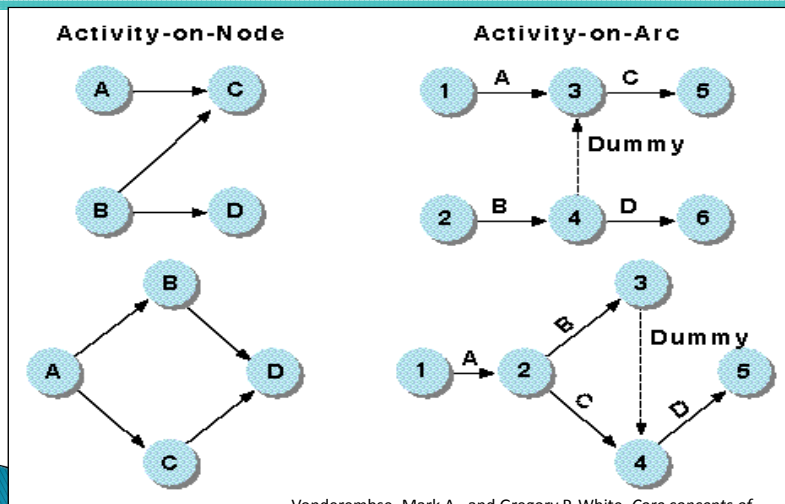


Activity-on-Arc Representation of Information System Project



Vonderembse, Mark A., and Gregory P. White. *Core concepts of operations management*. John Wiley & Sons Inc, 2003.

Dummy Activities for Activity-on-Arc Representation



Vonderembse, Mark A., and Gregory P. White. *Core concepts of operations management*. John Wiley & Sons Inc, 2003.

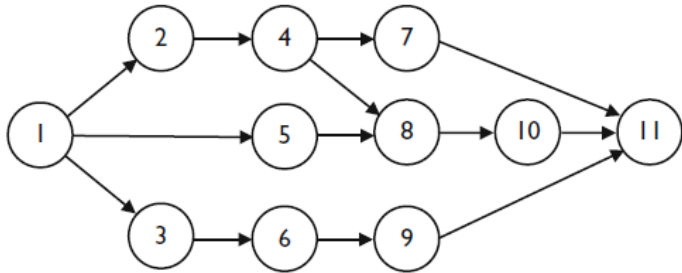
Exercise

| Activity | Predecessors |
|----------|--------------|
| 1 | — |
| 2 | 1 |
| 3 | 1 |
| 4 | 2 |
| 5 | 2 |
| 6 | 3 |
| 7 | 4 |
| 8 | 4, 5 |
| 9 | 6 |
| 10 | 8 |
| 11 | 7, 9, 10 |

Draw AON and AOA networks

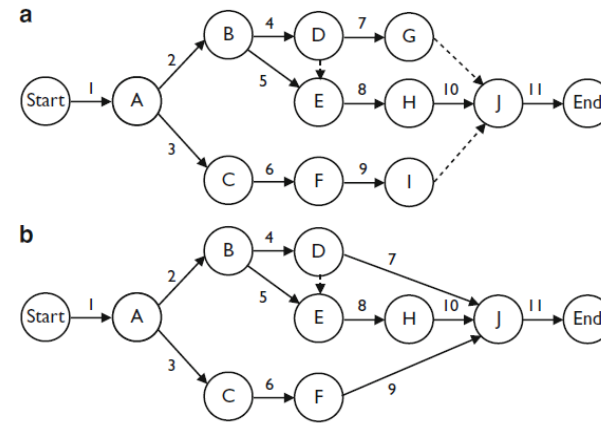
Vanhoucke, Mario. *Project management with dynamic scheduling*. Springer, Berlin, 2012.

Solutions for AON



Vanhoucke, Mario. *Project management with dynamic scheduling*. Springer, Berlin, 2012.

Solutions for AOA



What are the differences?

Vanhoucke, Mario. *Project management with dynamic scheduling*. Springer, Berlin, 2012.

CPM Objectives

- ▶ How long will the project take? (What is the minimum makespan?)
- ▶ How **early** can a particular activity be started?
- ▶ How far can an activity be **delayed** without increasing the total project duration?

CPM: Linear Programming (LP)

Min C_{n+1}

Objective Function

subject to

$$C_j - C_i \geq p_j \quad \forall (i, j) \in A$$

Precedence Relations

$$C_j \geq 0 \quad \forall j \in N$$

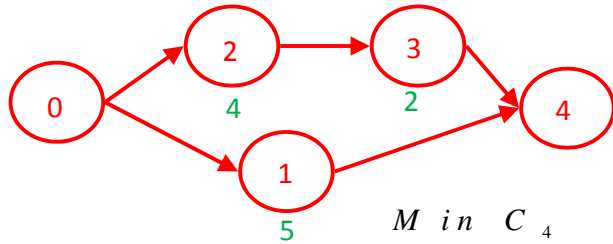
Non-negativity

p_j :Duration of activity j. (Parameter)

C_j :Completion time of activity j. (Decision Variables)

LP & IP Solvers: IBM ILOG CPLEX, GAMS, GUROBI

LP Exercise



Write down the LP for the project network given above.

$$\text{Min } C_4$$

s.t

$$C_1 - C_0 \geq 5$$

$$C_2 - C_0 \geq 4$$

$$C_3 - C_2 \geq 2$$

$$C_4 - C_1 \geq 0$$

$$C_4 - C_3 \geq 0$$

$$C_0 \geq 0$$

The Critical Path Method

- Calculating Earliest Start and Finish Times
 - $EF = ES + \text{activity duration}$
- Calculating Latest Start and Finish Times
 - $LS = LF + \text{activity duration}$

$$ES_i = \max\{EF_j \mid j \in PA_i\},$$

$$LF_i = \min\{LS_j \mid j \in SA_i\},$$

- PA (SA): preceding (succeeding activity)

Avoiding Late Completion

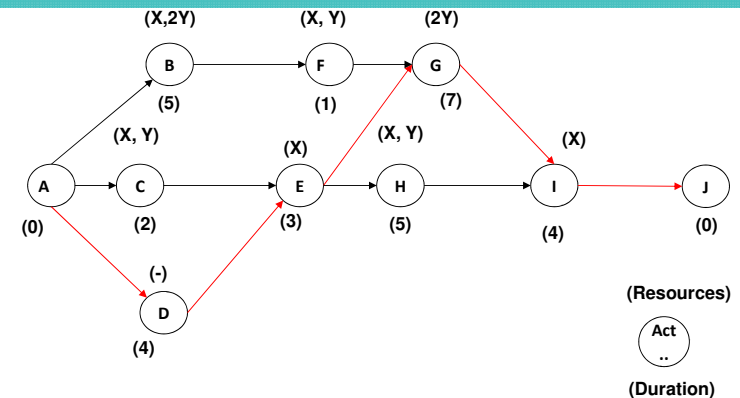
Total slack (TS) is the amount of time by which the completion time of an activity can exceed its earliest completion time **without delaying the project completion time**

$$\square \text{ Total Slack (TS)} = LF - EF$$

or

$$\square \text{ Total Slack} = LS - ES$$

Exercise



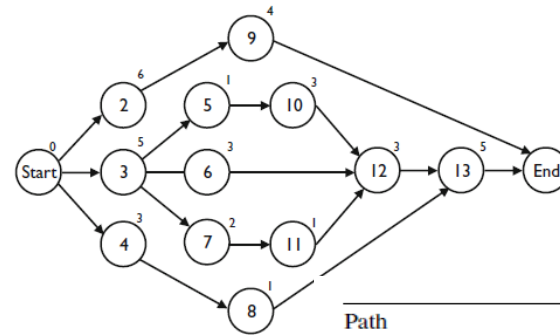
- What is the length of the critical path?
- What are the slack values for the activities?

CPM/ Slack Analysis

| Activity | ES | EF | LS | LF | Slack |
|----------|----|----|----|----|-------|
| A | 0 | 0 | 0 | 0 | 0 |
| B | 0 | 5 | 1 | 6 | 1 |
| C | 0 | 2 | 2 | 4 | 2 |
| D | 0 | 4 | 0 | 4 | 0 |
| E | 4 | 7 | 4 | 7 | 0 |
| F | 5 | 6 | 6 | 7 | 1 |
| G | 7 | 14 | 7 | 14 | 0 |
| H | 7 | 12 | 9 | 14 | 2 |
| I | 14 | 18 | 14 | 18 | 0 |
| J | 18 | 18 | 18 | 18 | 0 |

ES: Earliest Start EF: Earliest Finish
 LS: Latest Start LF: Latest Finish

CPM & Paths



What is the length of the **critical path**?

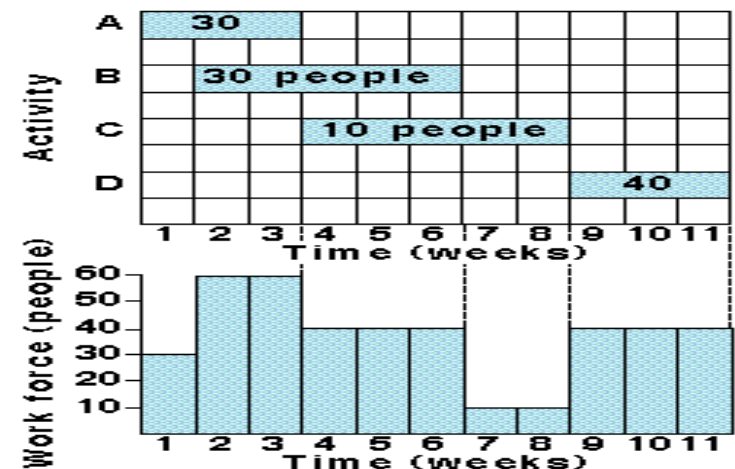
| Path | Duration |
|-------------------------------|----------|
| 1 → 2 → 9 → 14 | 10 |
| 1 → 3 → 5 → 10 → 12 → 13 → 14 | 17 |
| 1 → 3 → 6 → 12 → 13 → 14 | 16 |
| 1 → 3 → 7 → 11 → 12 → 13 → 14 | 16 |
| 1 → 4 → 8 → 13 → 14 | 9 |

Vanhoucke, Mario. *Project management with dynamic scheduling*. Springer, Berlin, 2012.

Quick Question

- ▶ Which of the following is late finish minus early finish or late start minus early start?
 - a. PERT
 - b. Slack
 - c. Critical Path
 - d. CPM
 - e. Crashing

Resource Requirements



Vonderembse, Mark A., and Gregory P. White. *Core concepts of operations management*. John Wiley & Sons Inc, 2003.

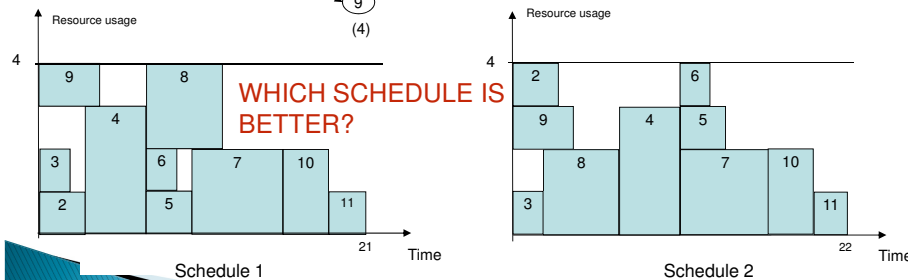
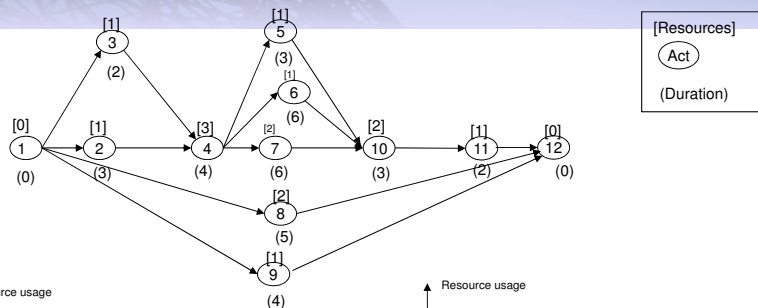
Resource Types

- ▶ **Renewable** : Machines, equipment and staff.
- ▶ **Nonrenewables** are consumable, i.e. materials.
- ▶ **Doubly constrained resources**: Limited availability **in every period** of the planning horizon and have constrained **total availability**.
- ▶ As project budgets control the consumption of money, both in every period and also over the duration of the complete project.

Resource Constrained Project Scheduling Problem (RCPSP)

- A deterministic single project scheduling problem.
- Constant resource availability and resource demands.
- Precedence relations among activities
- Minimizing the **project completion time (makespan)**.
- RCPSP is NP-hard. (Blazewicz et.al (1983)).

Resource Constrained Scheduling



Klein, Robert. *Resource-Constrained Scheduling Problems*. Springer US, 2000.

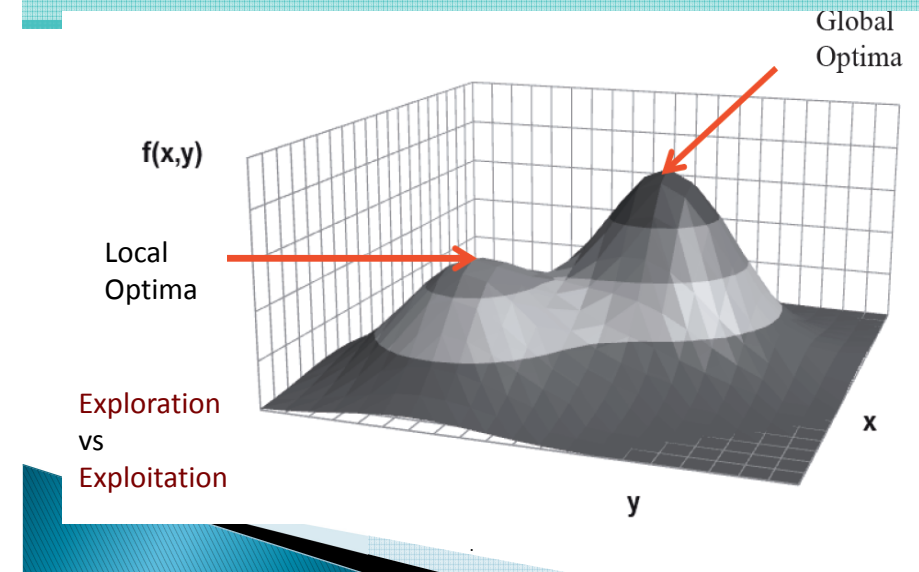
RCPSP- Solution Algorithms

- ▶ **Exact Solution**
 - Dynamic Programming.
 - Branch & Bound Method.
- ▶ **Heuristics (Approximate Solutions)**
 - **Priority-Rule Based Heuristics:**
 - The simplest & fastest heuristics.
 - Problem-Specific.
 - **Metaheuristics:**
 - High-level strategies for efficiently exploring search spaces.
 - Not problem-specific.

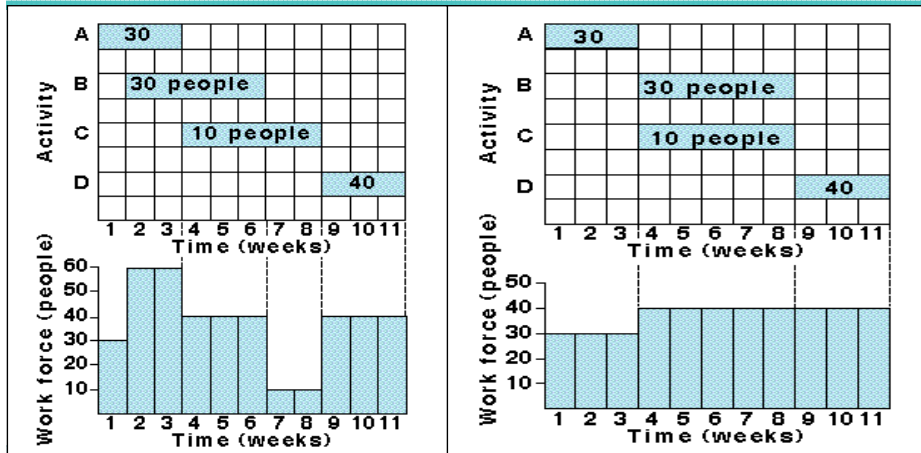
Metaheuristics

- Single Solution Based (Local Search)
 - Simulated annealing
 - Tabu search
- Population Based
 - Genetic Algorithms
 - Particle Swarm
- Demonstrate the quality of the solutions
- Optimality Gap (Find out **tight** Lower & Upper bounds)
- Demonstrate **the efficiency** of the algorithm (CPU time)
- Local Optima vs Global Optima

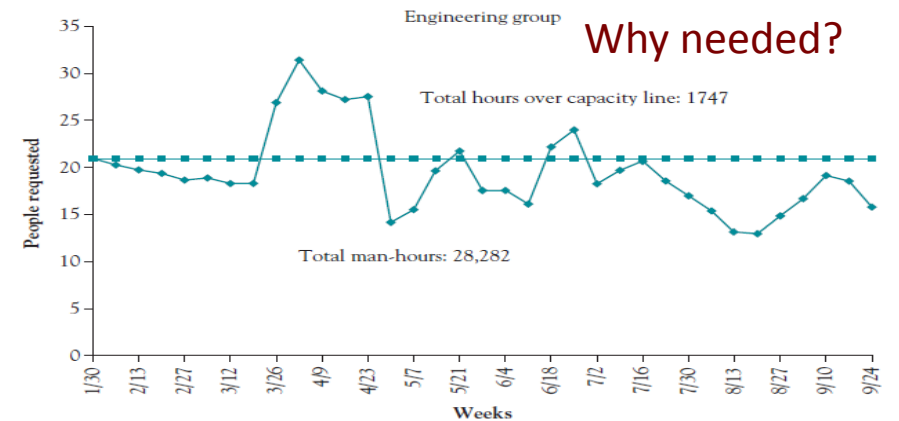
Metaheuristics & Exploration



Smoothed Work-Force Requirements



Resource Leveling

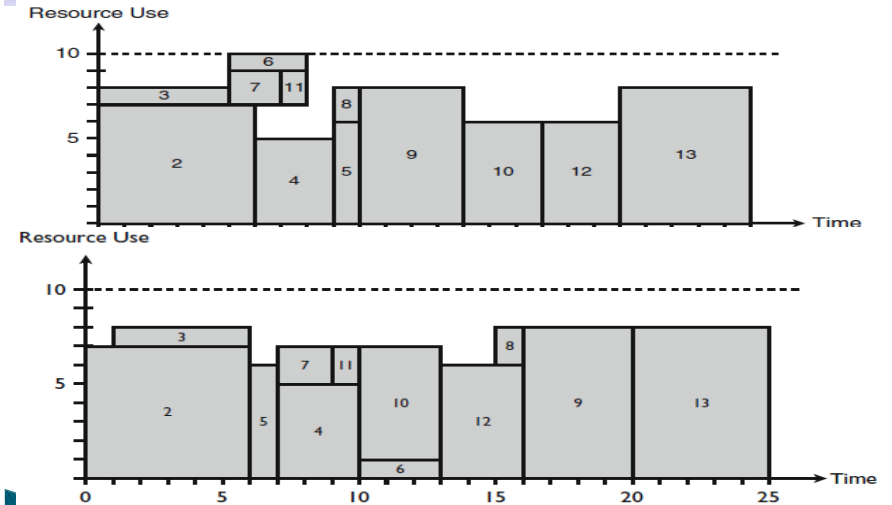


Resource Leveling

- *Resource-leveling* aims at the construction of a precedence and resource feasible schedule within a deadline with a resource use that is as level as possible.
- In order to avoid jumps from peaks to low resource demands, the total use of all resources needs to be balanced over the total schedule horizon.
- Minimize the **absolute** or **squared deviation** of the resource use from a given resource availability

Vanhoucke, Mario. *Project management with dynamic scheduling*. Springer, Berlin, 2012.

Resource Leveling

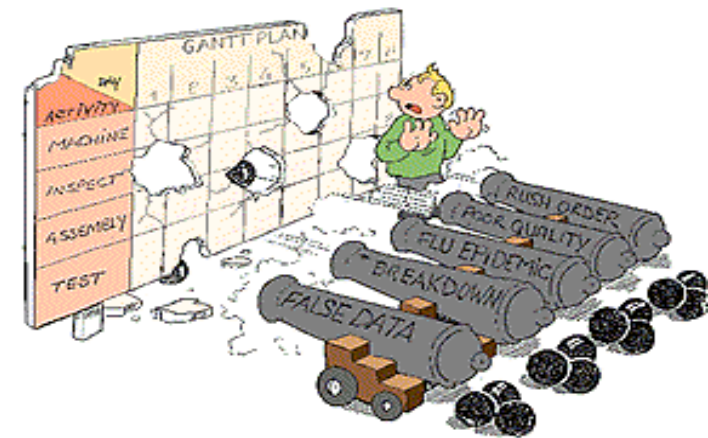


Vanhoucke, Mario. *Project management with dynamic scheduling*. Springer, Berlin, 2012.

Quick Question

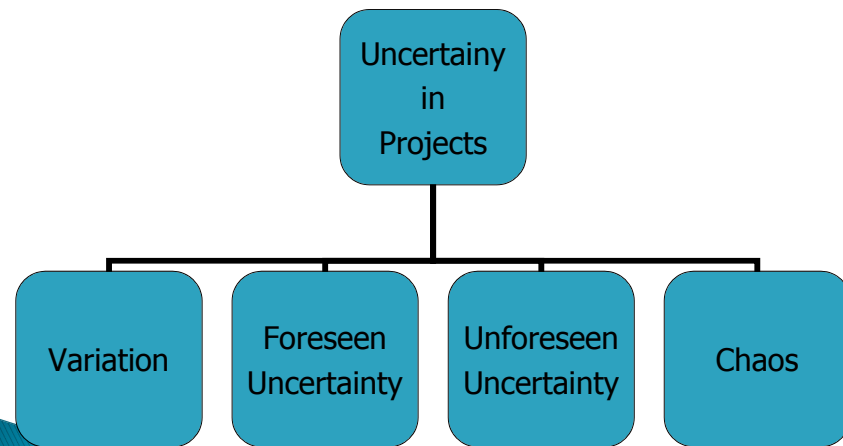
- ▶ The _____ approach to constrained **resource scheduling problems** is the feasible method of attacking large, nonlinear, **complex problems** that tend to occur in the **real world** of projects.
- Dynamic programming
 - Heuristic
 - Goal programming
 - Linear programming

Effects of Uncertainty

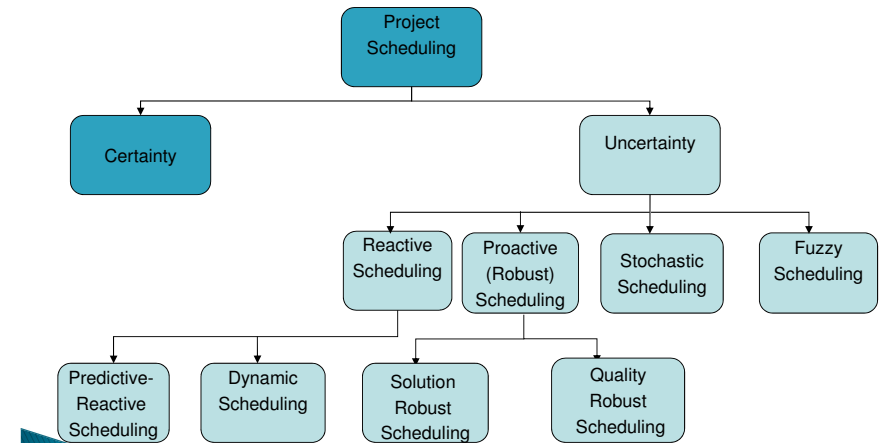


<http://www.uytes.com.tr/>

Uncertainty in Projects (De Meyer 2002)



Modeling Uncertainty in Projects

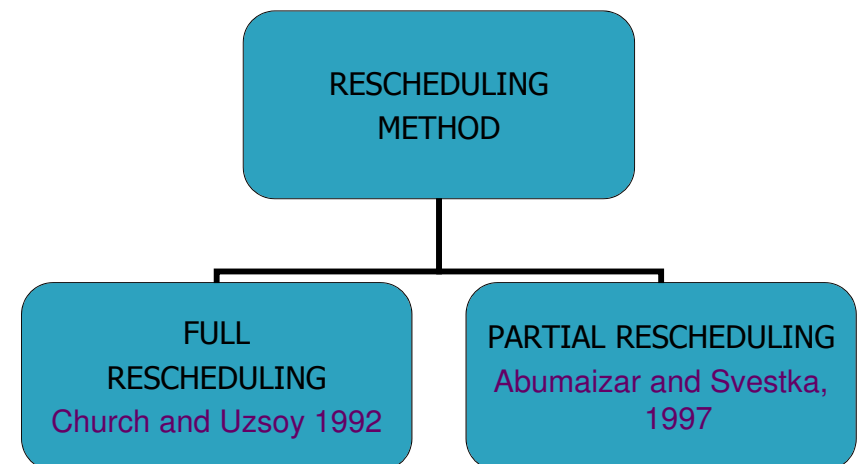


Herroelen and Leus (2005)

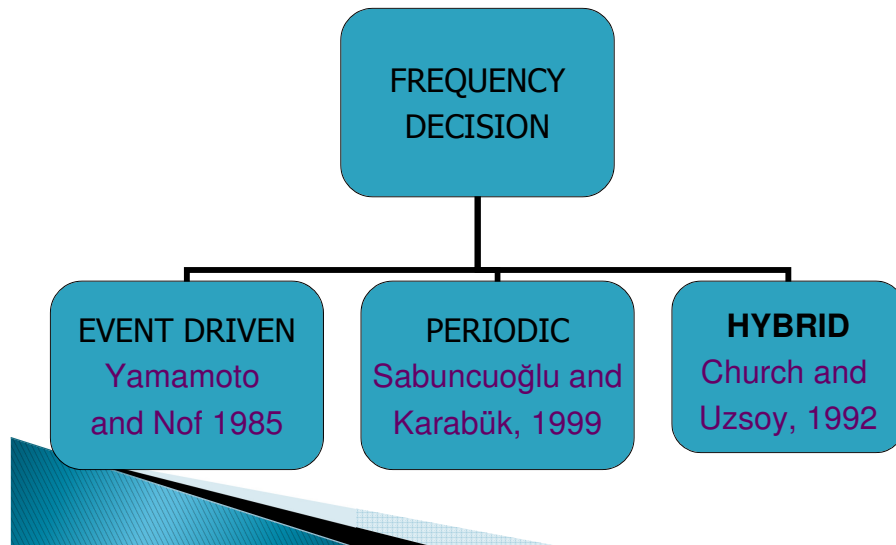
Reactive scheduling

- The baseline schedule is revised when an unexpected event occurs.
- **Dynamic scheduling:**
No **baseline schedule**.
- **Predictive-reactive scheduling:**
A baseline schedule.

Reactive Scheduling



Reactive Scheduling



Robustness Definitions

- ▶ Aims to generate schedules **insensitive** to disruptions.
- ▶ **Stability or Solution Robustness** :
 - The insensitivity of the **activity start times** with respect to disruptions.
- ▶ **Quality Robustness**:
 - The insensitivity of the **schedule performance (i.e completion time)** with respect to disruptions.
- ▶ The system performance remains under control even in the **worst-case conditions**.

CRITICAL CHAIN METHOD

- ❑ The critical chain method is the application of the **theory of constraints** (TOC) to project management.
- ❑ Eli Goldratt introduced TOC management philosophy in his book, "**The Goal**", in 1984. In 1997, he published "**Critical Chain**" and offered to apply TOC concepts in project management.
- ❑ TOC focuses on identifying & exploiting system **constraints** to improve performance of the **overall system**.

Critical Path vs. Critical Chain

- ❑ The critical path is the **longest path** in project network and determines the earliest completion time of the project.
- ❑ The critical chain considers both precedence and **resource relationships**. However, critical path considers only precedence dependencies.

Critical Chain Method Basics

Goldratt believes that when traditional approaches (CPM, PERT) are used:

- People have tendency to **overestimate** the activity times in order to **protect themselves** from **uncertainties**.
- The safety time created within the estimated times for each activity is misused.
 - **"Student syndrome"**: Since it is known that safety time is built into the estimates, people tend to delay starts or slow down.

Critical Chain Method

- **Safety factors** are eliminated from activities & aggregated **at the end** as a **project buffer**.
- **Aggressive time estimates** are used. This creates pressure to increase productivity.
- **Principle of aggregation** is utilized:
Standard deviation of the sum of a number of independent random variables is less than the sum of individual standard deviations.

Critical Chain Method

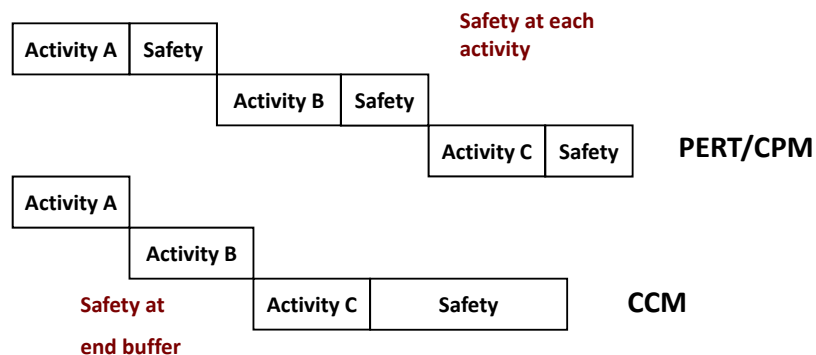


Figure : Safety Time Comparison (Rand,2000)

Buffer Types

- Buffers are used to protect against the uncertainty in the estimates of duration of tasks.
- There are three types of buffers:
 - Feeding buffers
 - Project buffers
 - Resource buffers

Critical Chain Method

- ❑ Multi-tasking, performing multiple tasks in the same time window, is discouraged.
 - Multi-tasking increases flow time of activities.
- ❑ CCM tries to eliminate due date driven behavior. Project milestones are eliminated. Instead of due dates for individual activities, ranges are communicated.
- ❑ CCM controls buffer usages to monitor project time performance.

Buffer Types

Buffers are used to protect against the uncertainty in the estimates of duration of tasks.

Project buffer:

- ❑ It is added to the **end of the critical chain** to protect the project delivery date.

Feeding buffers:

- ❑ They are added to **all paths feeding into the critical chain** to prevent the delays on those paths from affecting the start time of critical tasks.

Buffer Types

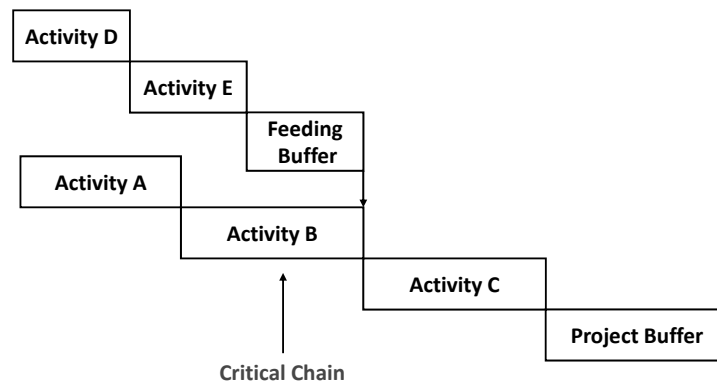
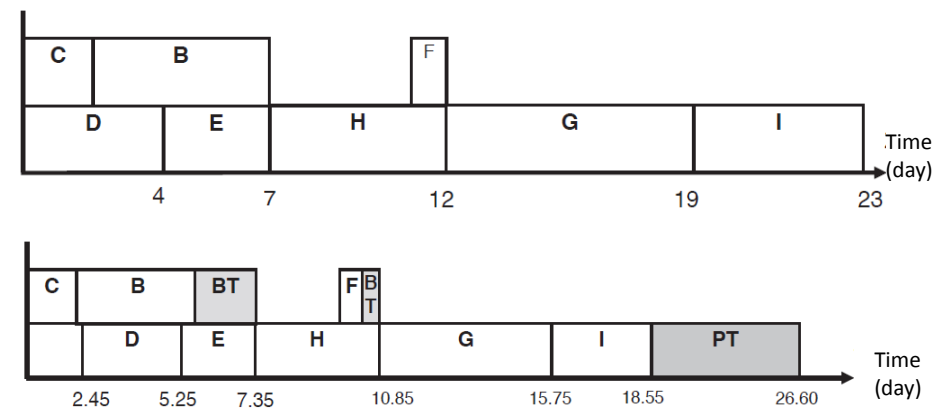


Figure : Project and Feeding Buffer (Rand,2000)

Buffer Management



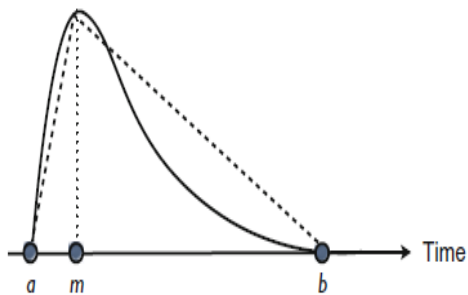
Critical Evaluation of CCM

- Aggressive time estimates will create **pressure to increase productivity**, this can lead to quality problems.
- CCM requires cultural change; multi-tasking and due date driven behavior will be eliminated.
- During project execution, the critical chain may change due to changes in **resource availability** and in buffer utilization.

Stochastic project scheduling

- Activity durations are **random variables**.
- Assumptions regarding **probability distributions**.
- No Baseline Schedule.
- Stochastic Programming.
- Expected Project Completion Time**

Program Evaluation & Review Technique: PERT



- PERT assumes that each activity duration is a **random variable** between two extreme values (i.e. a and b) & follows a **beta probability distribution**

PERT & Approximations

a - Optimistic time m - Most likely time b - Pessimistic time

Expected time: $t = \frac{a+4m+b}{6}$ Variance: $\sigma^2 = \left(\frac{b-a}{6}\right)^2$

| Activity | Optimistic a | Realistic m | Pessimistic b | t | σ |
|----------|--------------|-------------|---------------|---|------|
| 1 | 0 | 0 | 0 | 0 | 0 |
| 2 | 3 | 5 | 13 | 6 | 1.67 |
| 3 | 2 | 3.5 | 14 | 5 | 2 |
| 4 | 1 | 3 | 5 | 3 | 0.67 |
| 5 | 1 | 1 | 1 | 1 | 0 |
| 6 | 1 | 2 | 9 | 3 | 1.33 |
| 7 | 1 | 2 | 3 | 2 | 0.33 |
| 8 | 1 | 1 | 1 | 1 | 0 |
| 9 | 1 | 4 | 7 | 4 | 1 |
| 10 | 1 | 3 | 5 | 3 | 0.67 |
| 11 | 0.5 | 1 | 1.5 | 1 | 0.17 |
| 12 | 1 | 2.5 | 7 | 3 | 1 |
| 13 | 1 | 4 | 13 | 5 | 2 |
| 14 | 0 | 0 | 0 | 0 | 0 |

PERT & Outputs

PERT analysis provide the following information:

- The **expected entire project duration** and the critical path.
- The **probability** to complete the project within a specified deadline.
- The deadline before which the project can be completed with a certain probability.

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PERT& Assumptions

The **central limit theorem** states that distribution of the sum of the **independent** random variable is approximately normal when n is sufficiently large.

$$\mu_{CP} = \sum_{i \in CP} \mu_i, \quad \sigma_{CP}^2 = \sum_{i \in CP} \sigma_i^2$$

PERT Exercise

Probabilities: The probability that the example project has a total duration less than or equal to 20 time units equals

$$\begin{aligned} P(T \leq 20) &= P\left(\frac{T - 17}{3.07} \leq \frac{20 - 17}{3.07}\right) \\ &= P(z \leq 0.976) \\ &= 83.55\% \end{aligned}$$

Percentiles: The project duration T with a risk of exceeding of 10% is equal to the 90th percentile of the $N(17; 3.07)$ normal distribution and can be calculated as follows:

$$\begin{aligned} 90\% \text{ percentile} \rightarrow z = 1.28 &= \frac{T - 17}{3.07} \\ \rightarrow T &= 20.9 \text{ time units (e.g. weeks)} \end{aligned}$$

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Quick Question

- ▶ Which of the following is placed at the end of the project and includes all safety factors for critical path activities?
 - a. Project buffer
 - b. Feeding buffer
 - c. Resource buffer
 - d. Production buffer